

# Surface homeomorphisms and geometry

Math 732, Spring 2008, Chris Judge

**Description:** In this course we will develop geometric and analytic tools for studying a homeomorphism  $\phi$  from a surface  $S$  onto itself.

One approach to understanding a homeomorphism  $\phi : S \rightarrow S$  (up to homotopy) is to study the action of  $\phi$  on classes of closed curves on  $S$ . For example, each homeomorphism induces an action on the first homology group  $H^1(S, \mathbb{Z})$ . A more refined action involves *measured foliations* or *geodesic laminations*.

A second approach is to study the action of  $\phi$  on the space of constant curvature metrics (or conformal structures) on  $S$ . From this point of view, the group of (homotopy classes of) homeomorphisms can be regarded as a discrete group that acts properly discontinuously and isometrically on a finite dimensional manifold, the so-called *Teichmüller space*.

In the late 70's, Thurston observed that these two approaches are intimately related. The goal of this course is to explain Thurston's observation and some of its more recent consequences.

**Prerequisites:** The prospective student should have facility with basic topology (e.g. covering spaces and singular homology), and basic algebra (e.g. finitely presented groups). A familiarity with hyperbolic geometry will be helpful, but geometric facts will be presented/developed as needed. (Neither M533, M534, nor M731 are prerequisites for this course.)

**References:** No text book will be required. References will either be on hold in the library or will be distributed via e-mail. References include:

- *Automorphisms of surfaces after Nielsen and Thurston*. Notes of lectures by Andrew Casson taken by Steven Bleiler. London Mathematical Society Student Texts, 9. Cambridge University Press.
- *A primer on mapping class groups* by Benson Farb and Dan Margalit. <http://www.math.utah.edu/~margalit/primer>
- *Travaux de Thurston sur les surfaces* by Fathi, Laudenbach, and Poneru. Astérisque 66-67, Société Mathématique de France.
- *The geometry and topology of 3-manifolds* by William P. Thurston. <http://www.msri.org/communications/books/gt3m/PDF/index.pdf>
- *Geometry and spectra of compact Riemann surfaces* by Peter Buser, Birkhäuser.
- *Riemann surfaces, dynamics and geometry* by Curt McMullen. <http://abel.math.harvard.edu/~ctm/papers/index.html>