CHAPTER 1

Functions and Change

Section 1.1. What is a function?

A. Themes:

- Functions appear in everyday life.
- Functions may be represented by tables, graphs, and formulas.
- Qualitative properties of graphs have real world meaning.

B. Terminology.

- function
- value of a function
- horizontal intercept or x-intercept
- vertical intercept or y-intercept

C. Skills:

- Evaluate a function: e.g. Given formula, graph, or table, find $f(4)$.
  (Text: 10,13,15,17,25)
- Interpret value of function: e.g. $N(t)$ =number of monkeys escaping at time t. What does $N(5)$ mean?
  (Text: 1,3,5,10,25)
- Create or interpret graph based on experience: E.g. graph your level of physical activity during the day? Which of the following graphs might represent your hunger during a given day?
  (Text: 2,5,7,9,11)
- Interpret qualitative properties of graph: E.g. when does peak occur? What do the horizontal and vertical intercepts represent?
  (Text: 5,7,11,23)
- Determine range: E.g. let $F(x) = 3x^2 - 1$. Can either -2 or 4 be a value of F?
  (Text: 25)
- Use calculator to graph function given by a formula.
- Use calculator to generate table of values,

Section 1.2. Linear Functions

A. Themes:

- Graph of linear function is a line.
- (Nonvertical) linear equations determine linear function.
- Average rate of chane of linear function is constant.
- slope (average rate of change) measures steepness of graph.
- point-slope formula $y = y_0 + m(x - x_0)$.
- slope intercept formula $y = mx + b$ ($b = -mx_0 + y_0$) where b is y-intercept.

B. Terminology:

- linear equation
- linear function
- slope
- rise over run

C. Skills:

- Find the slope of a line.
- Find equation of a line given the slope and a point.
  (Text: 19)
- Given the y-intercept and the slope in a word problem, produce a formula for the function.
  (Text: 14,19)
- Find the equation of the line that passes through two points.
  (Text: 5,7,23,29)
- Find slope and y-intercept by converting a linear equation into $y = mx + b$ form.
  (Text: 1,3,29)
- Graph a linear function given by a formula.
  (Text: 9)
- Recognize qualitative properties of graph of linear function. E.g. do these two lines have the same slope? Is the slope of this line positive or negative? Which line has a larger vertical intercept?
  (Text: 11,19)
- Find slope and intercepts based on information given in a table.
  (Text: 19,23)
- Determine whether or not a table represents a linear function.
  (Text: 15,23)

Section 1.3. Rates of Change

A. Themes:

- average rate of change measures rate of change
- increasing and decreasing: direction of change
- concavity of graph: turning up/down

B. Terminology:

- average rate of change $\frac{f(b)-f(a)}{b-a}$
- increasing/decreasing
- concave up/down
- velocity/speed

C. Skills:

- Identify concavity given graph.
  (Text: 1,3,7)
- Identify intervals of increase/decrease of given graph.
  (Text: 7)
- Recognize positive/negative average rate change from experience.
  (Text: 11)
- Compute average rate of change over an interval of a function given by formula/table/graph.
  (Text: 13,15,16,19,21,27)

Section 1.4. Applications of Functions to Economics

A. Themes:

- Cost, revenue, and profit are functions of quantity and/or price.
- The depreciating value of capital equipment is a function of time.
- Quantity supplied or demanded is function of price.
- A two item budget is a level set of a linear fuction of two variables. Such a level set can be expressed as a linear equation in the plane.

B. Terminology:

- Cost
- Fixed costs
- Variable costs
- Revenue = price × quantity
- Profit = revenue − cost
- Break-even point
- Depreciation
- Supply
- Demand
- Equilibrium price
- Budget constraint

C. Skills:

- Determine variable/fixed costs from a given formula/table/graph for cost function.
  (Text: 2,3,4,7)
- Find formula for cost function given variable and fixed costs.
  (Text: 9,13)
- Determine the value of cost/revenue/profit function given formula/table/graph.
  (Text: 5)
- Determine price using $R = p \cdot q$ i.e. price is the slope of $R(q)$.
  (Text: 2)
- Find break-even point.
  (Text: 5,7)
- Determine when revenue is greater than cost and vice versa.
  (Text: 7)

- Find formula for depreciating capital equipment given initial value and rate at which it decreases.
  (Text: 15)
- Distinguish between demand curve table and supply curve table.
  (Text: 21)
- Interpret values from supply/demand curve table/graph.
  (Text: 21)
- Find equation for a linear demand curve.
  (Text: 19)
- Find the equilibrium price given two demand curves.
  (Text: 26,27)
- Determine the effect of a tax (absolute/percentage) on supply/demand curve and hence on equilibrium price.
  (Text: 26,27)

Section 1.5. Exponential Functions

A. Themes:

- Exponential functions describe a quantity whose change is proportional to current value. In other words, percentage growth is constant.
- In constrast, for linear functions absolute growth is constant.
- A discrete growth is approximated by a continuous one.
- Exponential functions have a wide variety of applications.

B. Terminology:

- base
- exponent
- exponential function
- exponential growth
- exponential decay
- percentage/relative rate of growth/decay
- initial value

C. Skills:

- Determine percentage (relative) rate of growth/function and/or initial value given a formula for an exponential function.
  (Text: 2)
- Interpret graphs of exponential functions. Compare with linear functions.
  (Text: 7)
- Find formula for exponential function given percentage/relative growth rate and initial value. Contrast with linear/absolute rate of growth. Determine value at another time.
  (Text: 3,5,17,25)
- Determine whether tabular data represents exponential function.
  (Text: 11,15,19)
- Find formula for exponential function given two data points e.g. from table or graph. *Nota Bene:* If t values of table are not incremented by 1, then divide exponent t by the increment.
  (11,19,23)

Section 1.5. pp. 82-89: Compound Interest and the Number e

A. Themes:

- Principal of account accruing compound interest is (discrete) exponential function.
- As compounding frequency tends to infinity, exponential function limits to $(e^r)^x$.
- This exponential function represents *continuous* percentage/relative growth. r is called the *continuous* rate of growth.

B. Terminology:

- compound interest
- annual interest rate

C. Skills:

- Find formula for pricipal after t years given annual interest rate compounded yearly, quarterly, monthly, weekly, daily, etc. (Text: 2,3)
- Find principal of above after given time period. (Text: 2,3)

Section 1.6. The Natural Logarithm

A. Themes:

- Introduce exponential function $x \to e^{rt}$ where r is the continuous growth rate.
- Natural logarithm is the inverse of $x \to e^x$.
- Natural logarithm converts multiplication to addition.
- Natural logarithm allows us to solve certain equations involving exponential functions e.g. how much time will it take savings account to grow to $1000?

B. Terminology:

- exponential functions with base e
- natural logarithm

C. Skills:

- Determine whether given exponential function represents decay/growth, continuous/discrete growth and what the percentage change is.
  (17,19,27,33,35)
- Determine the initial value of a given exponential function.
  (17,19)
- Solve an equation involving variable exponent using natural logarithm e.g. $4 = (1.06)^t$, $\frac{1}{2} = 8 \cdot e^{.06t}$.
  (Text: 3,5,9,11)
- Convert e^{rt} to a^t.
  (Text: 25,27)
- Convert a^t to e^{rt}.
  (Text: 33,35)

Section 1.7. Exponential Growth and Decay

A. Themes:

- Exponential growth and decay describes many phenomena.
- Exponential growth and decay are modeled by exponential functions.
- This section overlaps with Section 1.5.

B. Terminology:

- doubling time
- half-life
- future value
- present value

C. Skills:

- Find doubling time given the percentage/relative growth rate. (Text: 1,13)
- Find half-life given the percentage/relative decay rate. (*Nota Bene:* Text asks students to graph and estimate
  (Text: 3,16)
- Determine exponential function given half-life or doubling time. (Text: 4,29)
- Determine exponential function from given initial population and continuous percentage growth rate.
  (Text: 13)
- Find future value of exponential function given initial amount and continuous percentage growth/decay rate. E.g. principal in savings account compounded continuously. (Note: Could/should be done in 1.5/1.6)
  (Text: 5,29)
- Determine initial amount necessary to achieve given future value at given percentage growth/decay rate.
  (Text: 9)
- Determine time at which a 'given' exponential function achieves a specified value or a specified percentage of its initial value.
  (Text: 19,29)
- Find percentage growth/decay over specified period of time given the percentage growth decay over another period of time.
  (Text: 21,31) (Note: Could/should be done in Section 1.5)
- Determine the total value of several deposits made at different times but with a fixed interest rate. E.g. Make a \$1000 deposit today and a \$2000 deposit 1 year from now in a 5% componded

continuously account. How much available in 3 years?
(Text: 33,39) (Note: Could/should be done in Section 1.5/1.6)

Section 1.8. New Functions From Old

A. Themes:

- Compositions of functions.
- De-compositions of functions. (Towards Chain Rule/Substitution)
- Post/pre-composition with multiplication by a constant, $x \to c \cdot x$, results in vertcial/horizontal rescaling of the graph.
- Post/pre-composition with addition by a constant, $x \to x + c$, shifts graph c units up/down.

B. Terminology:

- compose, composition, composite

C. Skills:

- Find the composition of two given functions represented by formulas.
  (Text: 3,5)
- Given the graph of a function, find the graph of the result of post/pre-composing this function with a dilation/translation.
  (Text: 9,15,17)
- Given two functions represented by tables/graphs, determine values of the composition.
  (Text: 19, 21,23)
- Given a function, find a (nontrivial) de-composition.
  (Text: 7,30,31)

Section 1.9. Proportionality, Power Functions, and Polynomials

A. Themes:

- Inverse and direct proportionality.
- Power function is proportional to quantity raised to a power.
- Polynomials are linear combinations of positive powers.

B. Terminology:

- constant of proportionality
- inversely proportional
- power function
- polynomial
- coefficient
- leading term
- leading coefficient
- degree
- turning point

C. Skills:

- Determine whether an expression $f(x)$ can be re-expressed as $k \cdot x^p$, i.e a power function. Find k and p.
  (Text: 1,2,3,4,5,6,9,12)
- Describe a proportionality as a power function. E.g. circulation time of a mammal is proportional to the two-thirds power of its mass.
  (Text: 13,15,18,20)
- Determine the constant of proportionality given one data point. Then determine another value of the function. E.g. given that a mammal weighing 100 kg has circulation time of 10 seconds, find circulation time of mammal weighing 70 lbs.
  (Text: 18,20)
- Find degree and leading coefficient of given polynomial p. Identify power function that approximates $p(x)$ for large $|x|$.
  (Text: 21,23,25)
- Determine properties of a polynomial from its graph: the minimal possible degree of the polynomial and the sign of the leading coefficient.
  (Text: 35)

Section 1.9. pp. 88-92: Limits to infinity and end behavior

A. Themes:

- Compare growth/decay of functions at infinity.

B. Terminology:

- "in the long run", "in the long term"
- dominate
- end behavior

C. Skills:

- Compare power functions near infinity and determine which function dominates.
  (Text: 1,3,5,25)
- Compare power, polynomial, exponential, and logarithm functions near infinity and determine which dominates.
  (Text: 19,21,23,27)

CHAPTER 2

Rate of Change: The Derivative

Section 2.1. Instantaneous Rate of Change

A. Themes:

- Instantaneous rate of change is a limit over average rates of change (Example: instantaneous velocity)
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B. Terminology.

- instantaneous velocity
- instantaneous rate of change
- derivative at a point
- slope of a curve

C. Skills:

- Given table or graph, determine whether derivative is positive/negative given graph of function. Relate to increasing/decreasing.
  (Text: 14,17,23)
- Given a formula, estimate instantaneous velocity/rate of change by taking average velocity/rate of change over smaller intervals.
  (Text: 1,9,14)
- Given a graph, estimate derivative at a given point.
  (Text: 3,5,7)
- Given a graph, compare average rates of change over different intervals as well as instantaneous rates changes at endpoints.
  (Text: 5,7,17,19)
- Given a table, estimate instantaneous rate of change by computing the average of the average rates of change on left and right.
  (Text: 11,23)

Section 2.2. The derivative function

A. Themes:

- Positive/neagative tells us increasing/decreasing
- Derivative is a function.

B. Terminology:

- derivative function

C. Skills:

- Given graph of f, find the graph of the derivative f'.
  (Text: 1,3,7,9,11)
- Given table, estimate $f'(x)$.
  (Text: 5,15)
- Draw a graph of a function that satisfies certain information about its derivative. E.g. draw grpah whose derivative is positive for $x > 0$, negative for $x < 0$, and equal to zero for $x = 0$.
  (Text: 13)
- Given graph, estimate values of f and f'.
  (Text: 16)
- Given formula, estimate derivative of f at several places, and then try to guess formula for f'.
  (Text: 26,27)
- Given graph of f', find intervals of increase and decrease for f.
  (Text: 31)

Section 2.3. Interpretations of the Derivative

A. Themes:

- Everyday phenomena can be described with derivatives.

B. Terminology:

- local linear approximation
- Leibniz notation $\frac{dy}{dx}$ and $\frac{dy}{dx}$

C. Skills:

- Given 'real life' function, use experience to determine whether derivative is positive or negative.
  (Text: 1,20)
- Given 'real' function, give practical meaning of a statement about derivative.
  (Text: 1,3,5,11,13,15,21)
- Find units of derivative (e.g. miles/hr)
  (Text: 1,3,5,15,20)
- Estimate value of function given value of function and value of derivative at a nearby point ('Local linear approximation')
  (Text: 10,11,12,13)

Section 2.4. The Second Derivative

A. Themes:

- Second derivative determines concavity of graph and vice versa.
- Second derivative is rate of change of rate of change.

B. Terminology:

- second derivative
- Newton notation $f''(x)$
- Leibniz notation $\frac{d^2y}{dx^2}$

C. Skills:

- Based on graph, estimate whether second derivative is positive/negative/zero at a given point.
  (Text: 1,3,15,23)
- Based on graph, estimate intervals of positivity/negativity of second derivative
  (Text: 15,23)
- Based on table, estimate where second derivative is positive/negative
  (Text: 13,17,25)
- Interpret 'real-life' situation in terms of second derivative
  (Text: 19,21)

Section 2.5. Marginal Cost and Revenue

A. Themes:

- Cost and revenue functions are not always linear
- Comparison of marginal cost/revenue leads to profit maximization method

B. Terminology:

- marginal cost (rate of change in cost)
- marginal revenue (rate of change in revenue)
- maximizing profit

C. Skills:

- Apply local linear approximation to estimate value of cost/revenue
  (Text: 1,3,13)
- Based on graph, estimate marginal cost/revenue
  (Text: 2,11)
- Interpret cost/revenue based on grap (e.g. fixed costs; concavity)
  (Text: 9)
- Based on table, estimate marginal cost/revenue
  (Text: 8,10)
- Find quantity produced/sold at which profit is maximized
  (Text: 11,13)
- Interpret 'real-life' situation in terms of second derivative
  (Text: 19,21)

CHAPTER 3

Short cuts to differentiation

Section 3.1. Derivative formulas for powers and polynomials

A. Themes:

- Differentiation is linear (constant multiple rule; addition rule)
- Power rule
- Differentiation of polynomial

B. Terminology:

- Power rule: $\frac{d}{dx}x^p = p \cdot x^{p-1}$, $p \neq 0$
- Constant multiple rule: $\frac{d}{dx}kf(x) = k \cdot \frac{d}{dx}f(x)$
- Addition rule: $\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

C. Skills:

- Find derivative of constant function.
  (Text: 1)
- Find derivative of a power function.
  (Text: 7,37)
- Find derivative of a sum of power functions.
  (Text: 17,19,28,33,34,39)
- Find the value of the derivative (of a sum of power functions).
  (Text: 28,33,34,39)
- Interpret the value of the derivative graphically.
  (Text: 28, 39)
- Interpret the value of the derivative in a real life context.
  (Text:37)
- Given a function that is a sum of power functions, find the equation of the line tangent to the graph at a given point.
  (Text:39)

Section 3.2. Exponential and Logarithm Functions

A. Themes:

- Derivative of e^x is e^x
- Derivative of a^x is $\ln(a) \cdot a^x$
- Derivative of $\ln(x)$ is x^{-1}.

B. Skills:

- Find derivative of a sum of exponential functions and power functions.
  (Text: 1, 3, 11, 15, 27, 29, 31)
- Find derivative of a sum of logarithm functions and power functions.
  (Text: 19, 20, 33)
- Find and/or interpret the value of a derivative at a particular point.
  (Text: 29, 31)
- Find the equation of the line tangent to the graph of a function.
  (Text: 31, 33)

Section 3.3. The Chain Rule

A. Themes:

- The derivative of the composition of two functions is the derivative of the 'inside' function multiplied by the derivative of the 'outside' function (the latter being evaluated at the value of the inside function).

B. Terminology:

- The chain rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

C. Skills:

- Apply chain rule to find derivative of $f(x)^p$ for some function f and power p.
  (Text: 1, 5, 6, 27)
- Apply the chain rule to find the derivative of $e^{f(x)}$ for some function f.
  (Text: 15, 31, 37, 39)
- Apply the chain rule to find the derivative of $\ln(f(x))$ for some function $f(x)$.
  (Text: 21)
- Find and/or interpret the value of a derivative at a particular point.
  (Text: 31, 37)
- Find the equation of the line tangent to the graph of a function.
  (Text: 31)

Section 3.4. The Product and Quotient Rules

A. Themes:

- One can find the derivative of products and quotients of functions whose derivatives are known.

B. Terminology:

- The product rule:
  $\frac{d}{dx}(f(x) \cdot g(x)) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
- The quotient rule:

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$$

C. Skills:

- Find the derivative of the product of two functions.
  (Text: 5,7,16,17,36,37)
- Find the derivative of the quotient of two functions.
  (Text: 23,27)