CHAPTER 1

Functions

Section 1.1. What is a function?

A. Themes:

- Functions appear in everyday life.
- Functions may be represented by tables, graphs, and formulas.
- Qualitative properties of graphs have real world meaning.

B. Terminology.

- function
- value of a function
- horizontal intercept or x-intercept
- vertical intercept or y-intercept

C. Skills:

- Evaluate a function: e.g. Given formula, graph, or table, find $f(4)$.
  (Text: 10,13,15,17,25)
- Interpret value of function: e.g. $N(t)$ =number of monkeys escaping at time t. What does $N(5)$ mean?
  (Text: 1,3,5,10,25)
- Create or interpret graph based on experience: E.g. graph your level of physical activity during the day? Which of the following graphs might represent your hunger during a given day?
  (Text: 2,5,7,9,11)
- Interpret qualitative properties of graph: E.g. when does peak occur? What do the horizontal and vertical intercepts represent?
  (Text: 5,7,11,23)
- Determine range: E.g. let $F(x) = 3x^2 - 1$. Can either -2 or 4 be a value of F?
  (Text: 25)
- Use calculator to graph function given by a formula.
- Use calculator to generate table of values,

Section 1.2. Linear Functions

A. Themes:

- Graph of linear function is a line.
- (Nonvertical) linear equations determine linear function.
- Average rate of chane of linear function is constant.
- slope (average rate of change) measures steepness of graph.
- point-slope formula $y = y_0 + m(x - x_0)$.
- slope intercept formula $y = mx + b$ ($b = -mx_0 + y_0$) where b is y-intercept.

B. Terminology:

- linear equation
- linear function
- slope
- rise over run

C. Skills:

- Find the slope of a line.
- Find equation of a line given the slope and a point.
  (Text: 19)
- Given the y-intercept and the slope in a word problem, produce a formula for the function.
  (Text: 14,19)
- Find the equation of the line that passes through two points.
  (Text: 5,7,23,29)
- Find slope and y-intercept by converting a linear equation into $y = mx + b$ form.
  (Text: 1,3,29)
- Graph a linear function given by a formula.
  (Text: 9)
- Recognize qualitative properties of graph of linear function. E.g. do these two lines have the same slope? Is the slope of this line positive or negative? Which line has a larger vertical intercept?
  (Text: 11,19)
- Find slope and intercepts based on information given in a table.
  (Text: 19,23)
- Determine whether or not a table represents a linear function.
  (Text: 15,23)

Section 1.3. Rates of Change

A. Themes:

- average rate of change measures rate of change
- increasing and decreasing: direction of change
- concavity of graph: turning up/down

B. Terminology:

- average rate of change $\frac{f(b)-f(a)}{b-a}$
- increasing/decreasing
- concave up/down
- velocity/speed

C. Skills:

- Identify concavity given graph.
  (Text: 1,3,7)
- Identify intervals of increase/decrease of given graph.
  (Text: 7)
- Recognize positive/negative average rate change from experience.
  (Text: 11)
- Compute average rate of change over an interval of a function given by formula/table/graph.
  (Text: 13,15,16,19,21,27)

Section 1.4. Applications of Functions to Economics

A. Themes:

- Cost, revenue, and profit are functions of quantity and/or price.
- The depreciating value of capital equipment is a function of time.
- Quantity supplied or demanded is function of price.
- A two item budget is a level set of a linear fuction of two variables. Such a level set can be expressed as a linear equation in the plane.

B. Terminology:

- Cost
- Fixed costs
- Variable costs
- Revenue = price $\times$ quantity
- Profit = revenue $-$ cost

- Break-even point
- Depreciation
- Supply
- Demand
- Equilibrium price
- Budget constraint

C. Skills:

- Determine variable/fixed costs from a given formula/table/graph for cost function.
  (Text: 2,3,4,7)
- Find formula for cost function given variable and fixed costs.
  (Text: 9,13)
- Determine the value of cost/revenue/profit function given formula/table/graph.
  (Text: 5)
- Determine price using $R = p \cdot q$ i.e. price is the slope of $R(q)$.
  (Text: 2)
- Find break-even point.
  (Text: 5,7)
- Determine when revenue is greater than cost and vice versa.
  (Text: 7)
- Find formula for depreciating capital equipment given initial value and rate at which it decreases.
  (Text: 15)
- Distinguish between demand curve table and supply curve table.
  (Text: 21)
- Interpret values from supply/demand curve table/graph.
  (Text: 21)
- Find equation for a linear demand curve.
  (Text: 19)
- Find the equilibrium price given two demand curves.
  (Text: 26,27)
- Determine the effect of a tax (absolute/percentage) on supply/demand curve and hence on equilibrium price.
  (Text: 26,27)

Section 1.5. Exponential Functions

A. Themes:

- Exponential functions describe a quantity whose change is proportional to current value. In other words, percentage growth is constant.
- In constrast, for linear functions absolute growth is constant.
- A discrete growth is approximated by a continuous one.
- Exponential functions have a wide variety of applications.

B. Terminology:

- base
- exponent
- exponential function
- exponential growth
- exponential decay
- percentage/relative rate of growth/decay
- initial value

C. Skills:

- Determine percentage (relative) rate of growth/function and/or initial value given a formula for an exponential function.
  (Text: 2)
- Interpret graphs of exponential functions. Compare with linear functions.
  (Text: 7)
- Find formula for exponential function given percentage/relative growth rate and initial value. Contrast with linear/absolute rate of growth. Determine value at another time.
  (Text: 3,5,17,25)
- Determine whether tabular data represents exponential function.
  (Text: 11,15,19)
- Find formula for exponential function given two data points e.g. from table or graph. *Nota Bene:* If t values of table are not incremented by 1, then divide exponent t by the increment.
  (11,19,23)

Section 1.5. pp. 82-89: Compound Interest and the Number e

A. Themes:

- Principal of account accruing compound interest is (discrete) exponential function.
- As compounding frequency tends to infinity, exponential function limits to $(e^r)^x$.

- This exponential function represents *continuous* percentage/relative growth. r is called the *continuous* rate of growth.

B. Terminology:

- compound interest
- annual interest rate

C. Skills:

- Find formula for pricipal after t years given annual interest rate compounded yearly, quarterly, monthly, weekly, daily, etc.
  (Text: 2,3)
- Find principal of above after given time period.
  (Text: 2,3)

Section 1.6. The Natural Logarithm

A. Themes:

- Introduce exponential function $x \to e^{rt}$ where r is the continuous growth rate.
- Natural logarithm is the inverse of $x \to e^x$.
- Natural logarithm converts multiplication to addition.
- Natural logarithm allows us to solve certain equations involving exponential functions e.g. how much time will it take savings account to grow to \$1000?

B. Terminology:

- exponential functions with base e
- natural logarithm

C. Skills:

- Determine whether given exponential function represents decay/growth, continuous/discrete growth and what the percentage change is.
  (17,19,27,33,35)
- Determine the initial value of a given exponential function.
  (17,19)
- Solve an equation involving variable exponent using natural logarithm e.g. $4 = (1.06)^t$, $\frac{1}{2} = 8 \cdot e^{.06t}$.
  (Text: 3,5,9,11)
- Convert e^{rt} to a^t.
  (Text: 25,27)
- Convert a^t to e^{rt}.
  (Text: 33,35)

Section 1.7. Exponential Growth and Decay

A. Themes:

- Exponential growth and decay describes many phenomena.
- Exponential growth and decay are modeled by exponential functions.
- This section overlaps with Section 1.5.

B. Terminology:

- doubling time
- half-life
- future value
- present value

C. Skills:

- Find doubling time given the percentage/relative growth rate. (Text: 1,13)
- Find half-life given the percentage/relative decay rate. (*Nota Bene:* Text asks students to graph and estimate (Text: 3,16)
- Determine exponential function given half-life or doubling time. (Text: 4,29)
- Determine exponential function from given initial population and continuous percentage growth rate. (Text: 13)
- Find future value of exponential function given initial amount and continuous percentage growth/decay rate. E.g. principal in savings account compounded continuously. (Note: Could/should be done in 1.5/1.6) (Text: 5,29)
- Determine initial amount necessary to achieve given future value at given percentage growth/decay rate. (Text: 9)
- Determine time at which a 'given' exponential function achieves a specified value or a specified percentage of its initial value. (Text: 19,29)
- Find percentage growth/decay over specified period of time given the percentage growth decay over another period of time. (Text: 21,31) (Note: Could/should be done in Section 1.5)
- Determine the total value of several deposits made at different times but with a fixed interest rate. E.g. Make a $1000 deposit today and a $2000 deposit 1 year from now in a 5% componded

continuously account. How much available in 3 years?
(Text: 33,39) (Note: Could/should be done in Section 1.5/1.6)

Section 1.8. New Functions From Old

A. Themes:

- Compositions of functions.
- De-compositions of functions. (Towards Chain Rule/Substitution)
- Post/pre-composition with multiplication by a constant, $x \to c \cdot x$, results in vertcial/horizontal rescaling of the graph.
- Post/pre-composition with addition by a constant, $x \to x + c$, shifts graph c units up/down.

B. Terminology:

- compose, composition, composite

C. Skills:

- Find the composition of two given functions represented by formulas.
  (Text: 3,5)
- Given the graph of a function, find the graph of the result of post/pre-composing this function with a dilation/translation.
  (Text: 9,15,17)
- Given two functions represented by tables/graphs, determine values of the composition.
  (Text: 19, 21,23)
- Given a function, find a (nontrivial) de-composition.
  (Text: 7,30,31)

Section 1.9. Proportionality, Power Functions, and Polynomials

A. Themes:

- Inverse and direct proportionality.
- Power function is proportional to quantity raised to a power.
- Polynomials are linear combinations of positive powers.

B. Terminology:

- constant of proportionality
- inversely proportional
- power function
- polynomial
- coefficient
- leading term

- leading coefficient
- degree
- turning point

C. Skills:

- Determine whether an expression $f(x)$ can be re-expressed as $k \cdot x^p$, i.e a power function. Find k and p.
  (Text: 1,2,3,4,5,6,9,12)
- Describe a proportionality as a power function. E.g. circulation time of a mammal is proportional to the two-thirds power of its mass.
  (Text: 13,15,18,20)
- Determine the constant of proportionality given one data point. Then determine another value of the function. E.g. given that a mammal weighing 100 kg has circulation time of 10 seconds, find circulation time of mammal weighing 70 lbs.
  (Text: 18,20)
- Find degree and leading coefficient of given polynomial p. Identify power function that approximates $p(x)$ for large $|x|$.
  (Text: 21,23,25)
- Determine properties of a polynomial from its graph: the minimal possible degree of the polynomial and the sign of the leading coefficient.
  (Text: 35)