M119 Final Exam, Fall 2002

Last name:

First name:

Student ID: - -

Instructor:

Directions:

- This exam consists of 18 multiple choice questions and 2 partial credit problems.
- For each multiple choice question, you must circle the letter on this page that corresponds to the **best** choice. Each multiple choice question is worth 4.5 points.
- For each partial credit problem you must show all of your work in the space provided and circle your answers.

1) A B C D E
2) A B C D E
3) A B C D E
4) A B C D E
5) A B C D E
6) A B C D E
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13) A B C D E
14) A B C D E
15) A B C D E
16) A B C D E
17) A B C D E
18) A B C D E

Do not write below this line!

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
4.5	9	13.5	18	22.5	27	31.5	36	40.5	45	49.5	54	58.5	63	67.5	72	76.5	81

multi	———	points
	$\times 4.5 =$	
———	#19	
———	#20	
———	total	

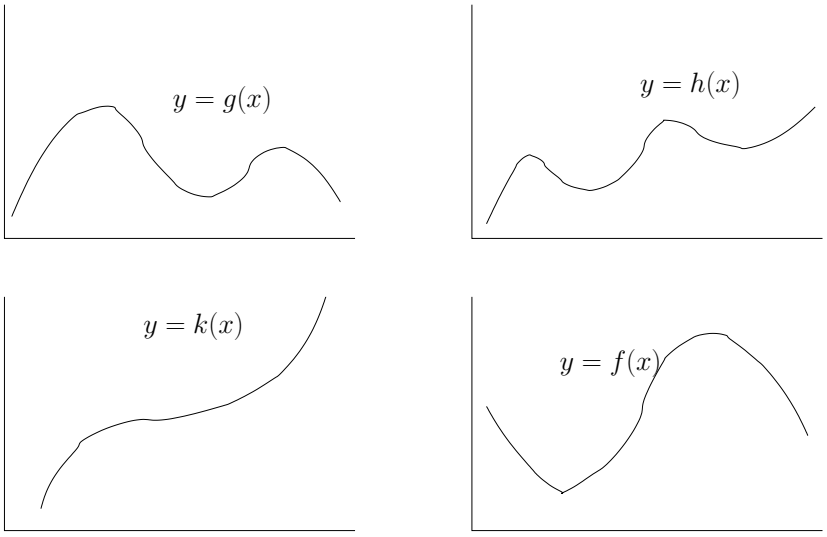

FIGURE 1. The graphs of the functions f, g, h, and k.

(1) Which one of the four functions pictured above has exactly 3 critical points?

A. f
B. g
C. h
D. k
E. None of the above

(2) In 1980, the world's population was 4.5 billion, and the population was projected to reach 7.8 billion by the year 2010. Assuming that this projection is true, then one can argue that the rate of growth of the world's population is approximately

A. 1.85 % per year.
B. 1.82 % per year.
C. 1.80 % per year.
D. 1.79 % per year.
E. None of the above.

(3) The value of the definite integral $\displaystyle\int_1^2 \frac{6}{z}\, dz$ is

A. 6
B. $6\ln(2)$
C. 12
D. $6\ln(1)$
E. none of the above.

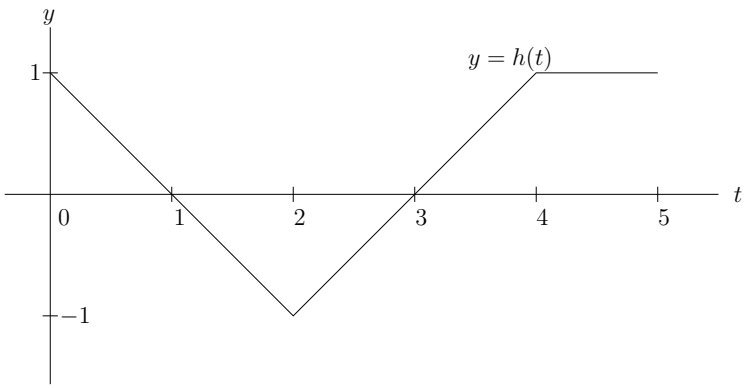

FIGURE 2. The graph of the function h.

(4) The graph of a function h is given above. The definite integral $\int_0^5 h(t)\ dt$ equals

A. 1.0
B. 2.5
C. 3.0
D. -2.0
E. None of the above

(5) The average value of the function $V(t) = t^2$ over the interval $[0, 6]$ is

A. 12
B. 6
C. 0
D. 18
E. None of the above.

(6) Some values of the function f are given in the following table.

x	0.1	0.3	0.5	0.9
$f(x)$	6.0	6.1	5.9	8.0

By using an average of estimates, one finds the best estimate of $f'(0.3)$ to be

A. $+0.20$
B. -0.10
C. $+0.05$
D. -0.25
E. None of the above

(7) The following table gives values for a function p.

t	3	6	9	12
$p(t)$	5	15	45	135

The function p is

A. a polynomial of degree at least 2
B. a linear function
C. a power function whose power equals $\frac{1}{2}$
D. an exponential function
E. the natural logarithm

(8) The average rate of change of $P(t) = 5t^3 + 9$ between $t = -2$ and $t = 5$ is

A. -97
B. 95
C. -93
D. 99
E. None of the above

(9) A trust receives a continuous income stream at the rate of $1,000$ dollars per year t years after the trust is created. The income earns 7% annual interest, compounded continuously. Ten years after its creation, the trust contains approximately

A. $ 14,482
B. $ 7,191
C. $ 10,000
D. $ 12,013
E. None of the above is a reasonable approximation.

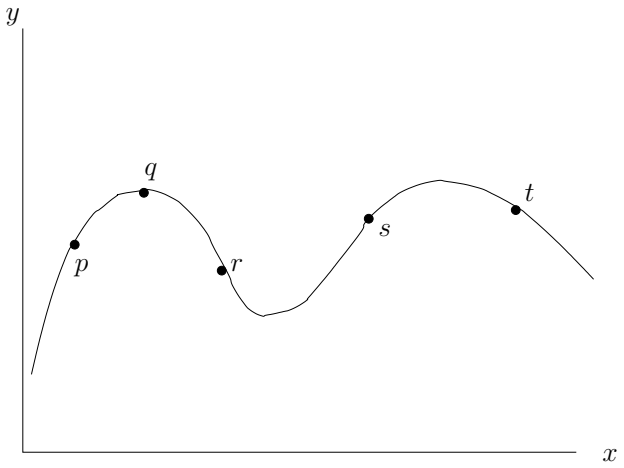

FIGURE 3. The points p, q, r, s, t on the graph of a function.

(10) Consider the function whose graph appears above. Both $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ are negative at the point

A. r
B. q
C. t
D. s
E. p

(11) An antiderivative for $x^{\frac{1}{2}}$ is

A. $\dfrac{3}{2}x^{\frac{3}{2}}$
B. $\dfrac{1}{2}x^{-\frac{1}{2}}$
C. $\dfrac{2}{3}x^{\frac{3}{2}}$
D. $-\dfrac{1}{2}x^{-\frac{1}{2}}$
E. None of the above.

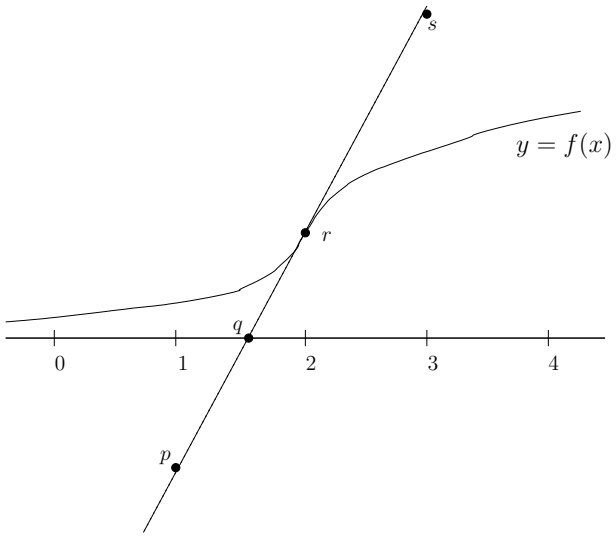

FIGURE 4. Points on a line tangent to the graph of f.

(12) Suppose the function f graphed above satisfies $f(2) = 1$ and $f'(2) = 3$. Then the coordinates of

A. the point p are $x = 1$ and $y = -5$
B. the point s are $x = 3$ and $y = 14$
C. the point r are $x = 2$ and $y = 3/2$
D. the point q are $x = 5/3$ and $y = 0$
E. None of the above

(13) A rumor spreads among a group of 900 people. The number of people who have heard the rumor t hours after the rumor started is modeled by

$$N(t) = \frac{900}{1 + 899e^{-.4t}}$$

The rumor spreads the fastest approximately

A. 17 hours after the rumor starts.
B. 14 hours after the rumor starts.
C. 21 hours after the rumor starts.
D. 9 hours after the rumor starts.
E. None of the above.

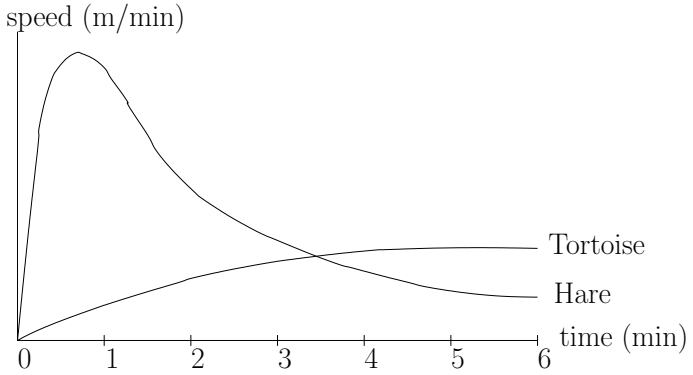

FIGURE 5. The speeds of the Hare and the Tortoise t minutes after the start of the race.

(14) The Hare and the Tortoise have a race. Their respective **speeds**, graphed against time, are given above. Which one of the following statements is true?

A. 3 minutes after the start of the race, the Tortoise has traveled farther than the Hare.
B. The Hare's minimum speed occurs at 6 minutes after the start.
C. The Tortoise catches up to the Hare approximately 3.5 minutes after the start.
D. 6 minutes after the start of the race, the Hare has traveled farther than the Tortoise.
E. None of the above statements are true.

(15) The derivative of $\ln\left(\frac{x}{k}\right)$ equals

A. $\frac{1}{x} + k$
B. $\frac{1}{x}$
C. $\frac{1}{kx}$
D. $\frac{k}{x}$
E. None of the above.

(16) Some values of the functions f, g, and their derivatives, f', g', are provided in the following table.

x	-2	-1	0	1	2
$f(x)$	2	-2	1	-1	0
$f'(x)$	-2	0	-1	2	1

x	-2	-1	0	1	2
$g(x)$	1	2	-1	0	-2
$g'(x)$	0	-2	1	2	-1

The derivative of $f(g(x))$ at $x = 2$ equals

A. 1
B. -4
C. 0
D. -1
E. 2

(17) If $f(t) = e^{-bt^2}$, then $f''(t)$, the **second derivative** of f, equals

A. $-2be^{-bt^2} + 4bte^{-bt^2}$
B. $-2be^{-bt^2}$
C. $-2be^{-bt^2} + 4b^2t^2e^{-bt^2}$
D. $-be^{t^2}$
E. none of the above.

(18) The indefinite integral $\int x(x^2+1)^{-2}\,dx$ equals

A. $(x^2+1)^{-3} - 2x^2(x^2+1)^{-2} + C$
B. $2 \cdot \ln(x^2+1)$
C. $-\frac{1}{2}(x^2+1)^{-1} + C$
D. $-(x^2+1)^{-2} + C$
E. None of the above.

Partial credit problems.
Show all work and reasoning! Circle your answers!

(19) *(Each part is worth 2 points.)*

The energy, E, expended by a bird per day depends on the number of hours, t, spent foraging for food. For one species of bird, biologists have found that

$$E(t) \;=\; 2t + \frac{8}{t^2}.$$

A. Find the derivative $E'(t)$.

B. Find all critical points of E.

C. Use the first or second derivative test to determine which critical points are local maxima and which are local minima.

D. Give the two endpoints of the interval on which t varies. (In other words, state the domain of E.) Briefly explain your answer.

E. Find the time spent foraging that minimizes the amount of energy expended by the bird.

(20) *(Each part is worth 3 points.)*

The marginal cost, $C'(q)$, of producing q tons of limestone is $q + .002 \cdot q^2$ dollars.

A. Using 4 equal subintervals, give the best upper estimate of the total change in cost caused by raising the production level from 40 to 80 tons.

B. Find an antiderivative of the function $C'(q) = q + .002 \cdot q^2$.

C. Given that the fixed costs $C(0) = 20,000$, find the total cost of producing 40 tons of limestone.